Advances in Astronomical Image Addition and Subtraction

Barak Zackay
Weizmann Institute of Science
Collaborators: Eran Ofek and Avishay Gal-Yam

A little bit about me

- Long term goal - Improving the state of the art observing techniques in astrophysics.
- Tools - Statistics and Algorithms

Major past works

- ZOGY image subtraction (this talk)
  - Employed in ZTF, BlackGEM, LSST (and many more)
- FDMT - algorithm for detecting FRBs
  - Used in ASKAP, discovered dozens of FRBs
- GW - BBH detection - an independent pipeline (with Tejaswi Venumadhav, Matias Zaldarriaga +…)
  - Doubled the number of detected binary black holes on the LIGO-VIRGO open data
  - Influenced other pipelines to increase sensitivity across the field.

Current work trajectories

- An algorithm to detect pulsars in binary systems (expected improvement: x3 sensitivity, **Hiring**)
  - **More than 10 applications in astrophysics alone**
- An algorithm to detect gamma-ray pulsars (I expect as much as x5 improvement in sensitivity, **Hiring**)
- Employing GW detection techniques for detecting transiting planets in Kepler data **(Oryna Ivashtenko)**
- Developing a new method (imaging + statistic + algorithms) for high contrast imaging of exoplanets **(Dotan Gazith)**
- GW astronomy - (detecting sGRBs, detecting precessing events, reducing control noise in LIGO**, Jonathan Mushkin, Hiring**)

Why am I telling you this?

- **Contact me** if you are a talented student/postdoc and you are:
  - Looking for a position (or to collaborate)
  - Interested in novel statistics and algorithms.
  - looking to make a dramatic impact in astrophysics.
- Come to me, and you will: **learn** a lot, **Invent and** have all **legitimacy** and **resources** required to **revolutionize** a field in astrophysics.
- There are special postdoc scholarships for Brazilian citizens in Weizmann!

I am hiring.

Advances in Astronomical Image Addition and Subtraction

Barak Zackay
Weizmann Institute of Science
Collaborators: Eran Ofek and Avishay Gal-Yam

Notation and image model

- Image model:

$$M_j = P_j \otimes T + \epsilon_j$$

- M_j - j'th measurement
- T - True sky.
- P_j - j'th PSF
- ϵ_j - additive, white Gaussian noise.

Problem definition

- What do we want when adding/subtracting images?
  - Maximum sensitivity for all astrophysical measurements.
  - Reliable detection of sources/transients.
  - No human involvement.
- What we should not care about?
  - Image quality metrics (image SNR, sharpness, resolution, seeing, …)

Stages in the pipeline

- Image calibration (flat fielding, bias and gain).
- Finding an astrometric solution.
- Image alignment (shifting, rotating, removing distortion and resampling).
- PSF, background and zero-point estimation.
- Identifying bad pixels and particle hits.
- Image coaddition/subtraction
- Source/transient detection
- …

Coaddition - commonly used methods

Weighted addition methods:
(Annis et al., 2014 Jiang et al., 2014)

$$S = \sum_j \alpha_j M_j$$

PSF homogenization
(Desai et al.,)

$$S = \sum_j \alpha_j K_j \otimes M_j$$

$$M_j \otimes K_j \approx M_{ref}$$

Speckle imaging methods:
Lucky imaging
Speckle interferometry

Problems with existing coaddition methods

- No argumentation or reasoning.
- Reduced sensitivity (5%-25% decrease in survey speed!).
- “No coaddition method is good for all applications”.
  - Trade-off between resolution and depth.
- Images with “bad” atmospheric conditions are discarded.
- Some coaddition methods involve regularized deconvolution (“PSF homogenization”)
  - Unstable, introduces spatial correlations and slow.
  - Unclear what further signal processing steps should follow.
  - Even less sensitive than weighted addition!

Our approach

- Go by the book:
  - Define the simplest statistical task - point source detection
  - Find it's optimal statistic using Neyman-Pearson.
  - Extend the solution to all tasks (if possible).
  - Analyze the solution's behavior when adding realistic complexities to the statistical model.
  - Apply corrections where needed.

Optimal source detection

- Statistical task - detecting point sources.

$$\mathcal{H}_0 : \; M_j = \epsilon_j$$

$$\mathcal{H}_1 : \; M_j = \delta_p \otimes P_j + \epsilon_j$$

$$S = \frac{\mathcal{P}(\{M\}|\mathcal{H}_1)}{\mathcal{P}(\{M\}|\mathcal{H}_0)} = \cdots = \sum_j \frac{\overleftarrow{P_j} \otimes M_j}{\sigma_j^2}$$

- S is the analogue of a match filtered image
    - Has correlated noise!
    - Does not fit our image model.

Sufficient statistic

- Statistical task - detecting any other source.

$$\mathcal{H}_0 : \; M_j = \epsilon_j$$

$$\mathcal{H}_1 \; : \; M_j = T(\theta) \otimes P_j + \epsilon_j$$

$$S = \frac{\mathcal{P}(\{M\}|\mathcal{H}_1)}{\mathcal{P}(\{M\}|\mathcal{H}_0)} = \cdots = \overleftarrow{T(\theta)} \otimes \sum_j \frac{\overleftarrow{P_j} \otimes M}{\sigma_j^2}$$

- Same trick works for **any** measurement.
- S is still not simple to use.
- Does not fit our image model.

Proper coaddition

- In fact, any two simple hypotheses about T could be tested using S.
- If S is a matched filtered image, can we find it's "original" image?

$$\hat{R} = \frac{\sum_j \frac{\overline{\hat{P}_j} \hat{M}_j}{\sigma_j^2}}{\sqrt{\sum_j \frac{|\hat{P}_j|^2}{\sigma_j^2}}} \qquad \hat{P}_r = \sqrt{\sum_j \frac{|\hat{P}_j|^2}{\sigma_j^2}}$$

Properties of the new coadd image

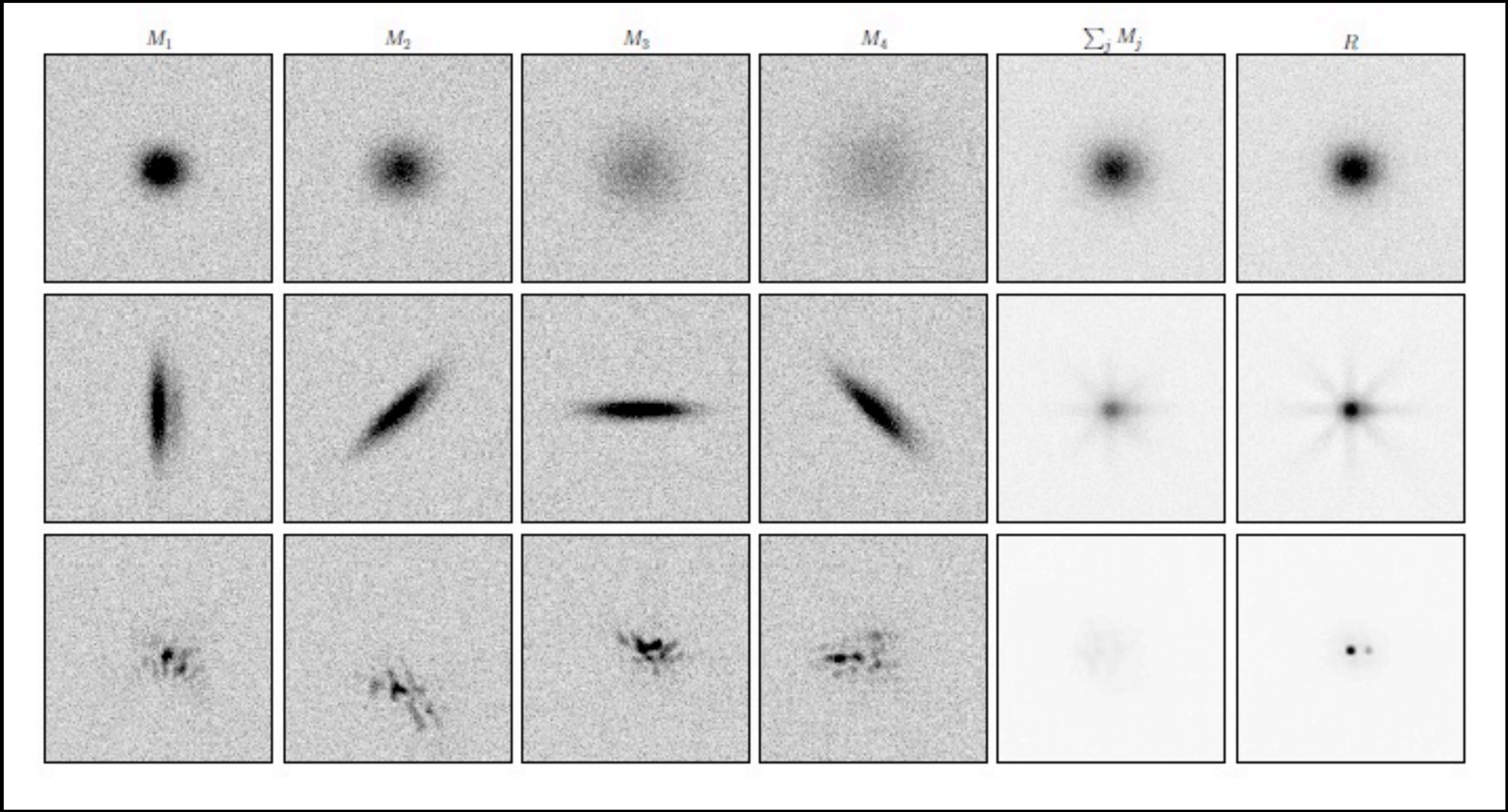
- Optimal for **all** decisions and measurements.
  - Assumes **known** PSFs and white Gaussian noise.
  - **Sufficient statistic** - Original data is redundant.
- 5%-25% more survey speed relative to weighted summation.
  - Even better relative to PSF homogenization.
- Numerically stable.
- Local - Can handle spatially changing PSF’s.
- Indistinguishable from a regular image.

Results

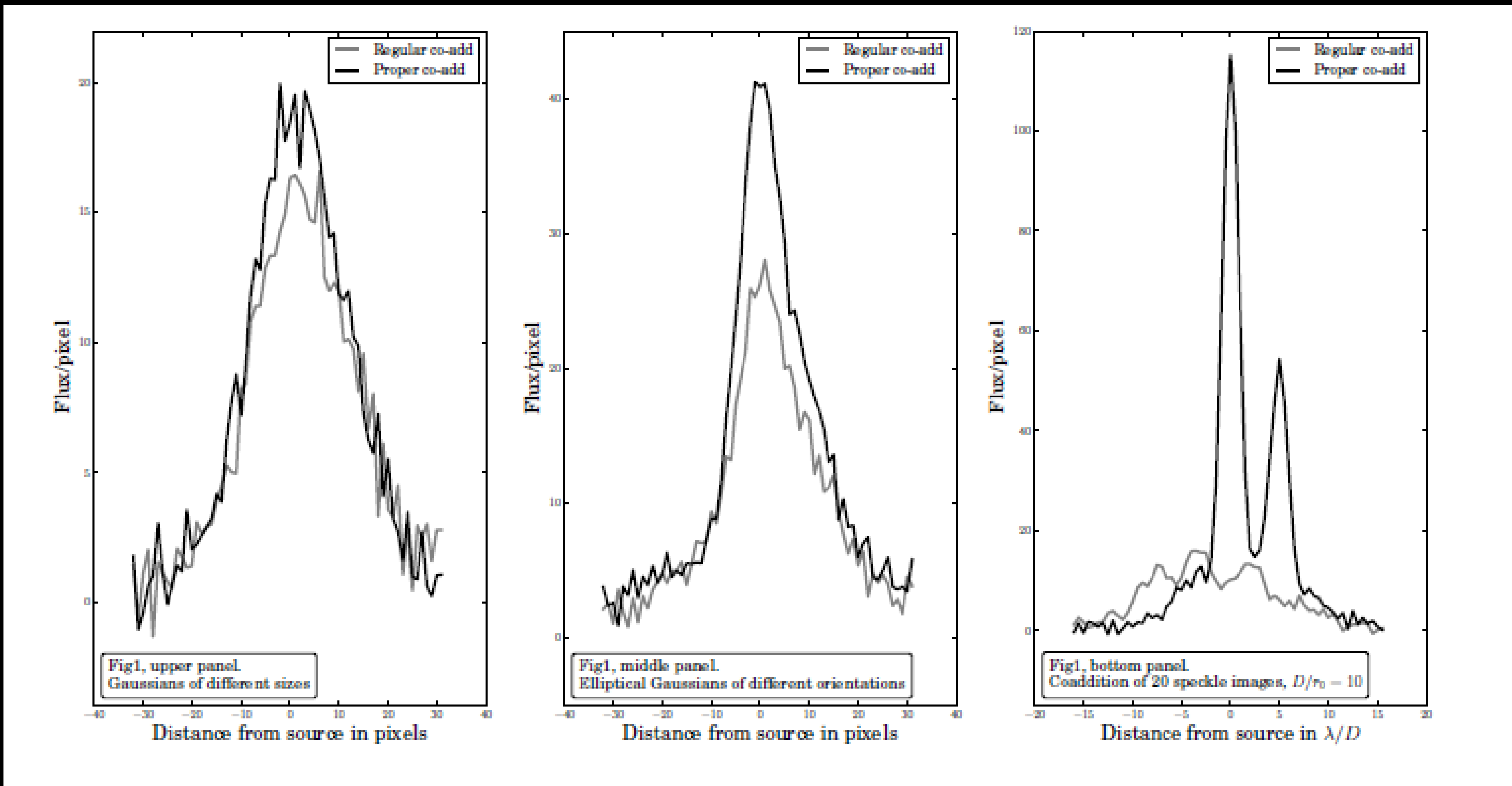
Simulated images of a binary star

Direct coadd

Proper coadd



Results



Regular co-add
Proper co-add
Flux/pixel
Fig1, upper panel.
Gaussians of different sizes
Distance from source in pixels
Regular co-add
Proper co-add
Flux/pixel
Fig1, middle panel.
Elliptical Gaussians of different orientations
Distance from source in pixels
Regular co-add
Proper co-add
Flux/pixel
Fig1, bottom panel.
Coaddition of 20 speckle images, $D/r_0 = 10$
Distance from source in λ/D

Future prospects - coaddition

- Deployment -One coadd (per color) to summarize a survey
  - Makes all sky surveys compact and distributable for everyone.
  - provides 5%-25% more survey speed.
- Super resolution - the sufficient statistic version
  - Summarizing sets of under-sampled images (Important for ULTRASAT and JWST)

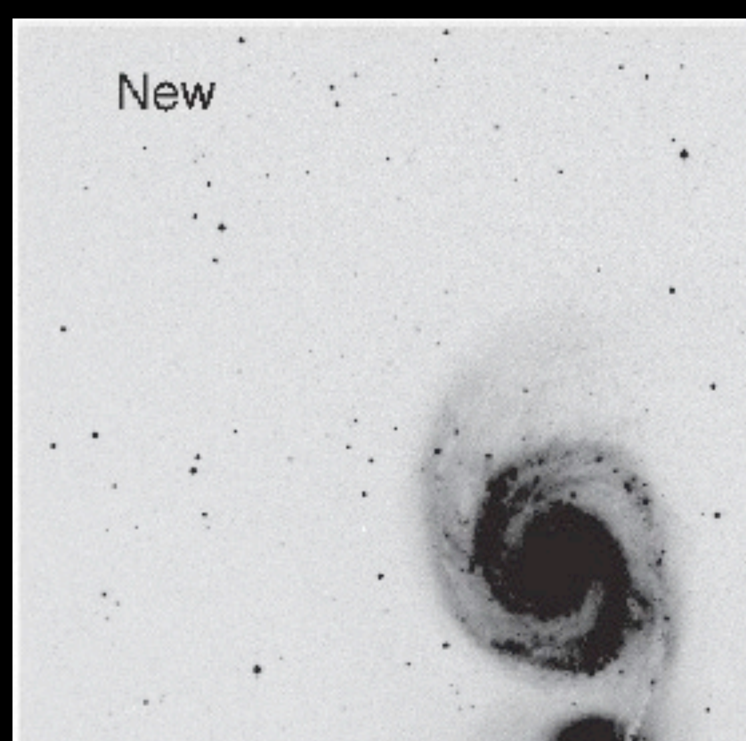
Sketch of the Super Resolution Project

Image formation

Impinging field on telescope aperture

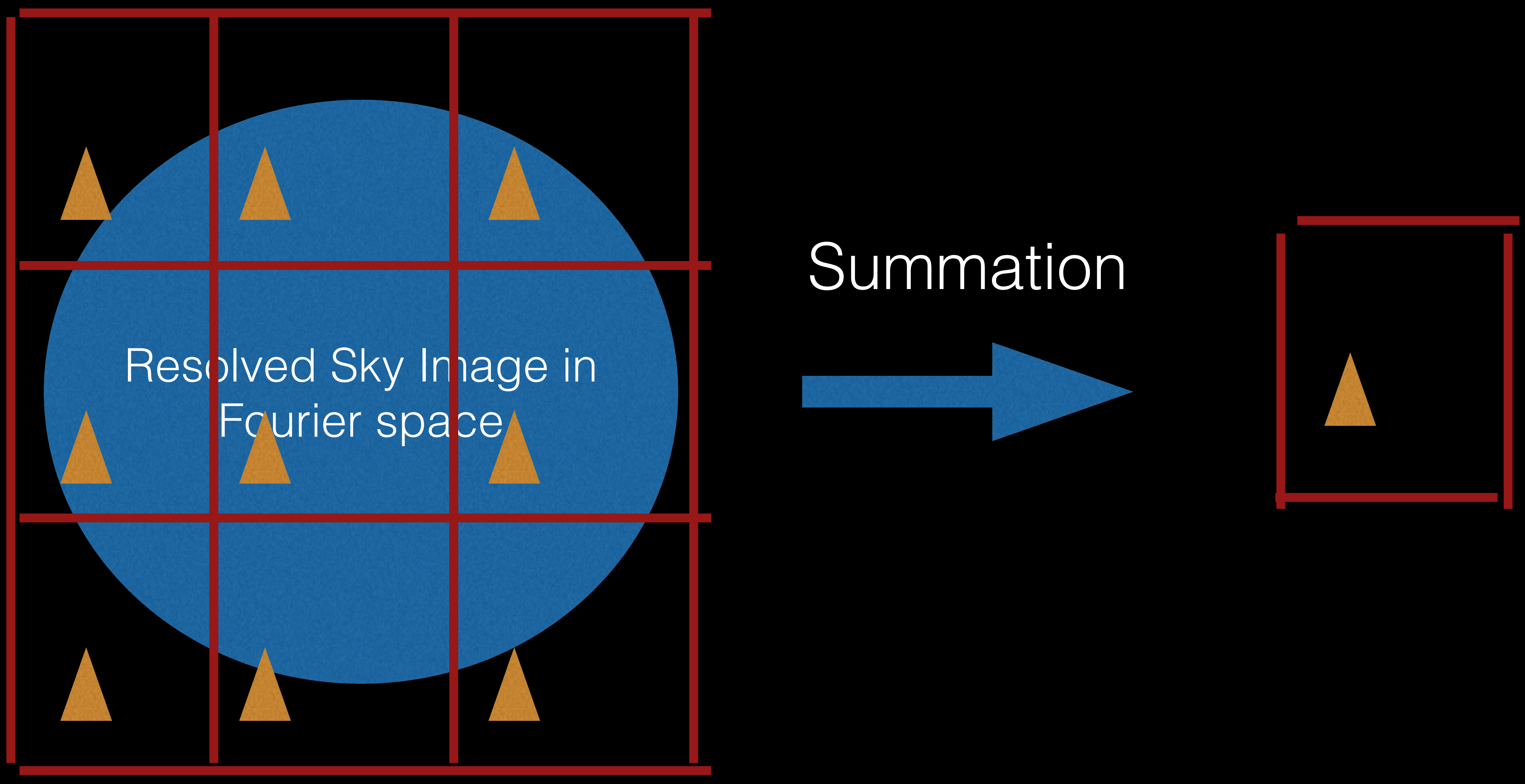
Fourier transform

Electric Field on detector



Absolute value squared
Convolution with pixel response
Sampling

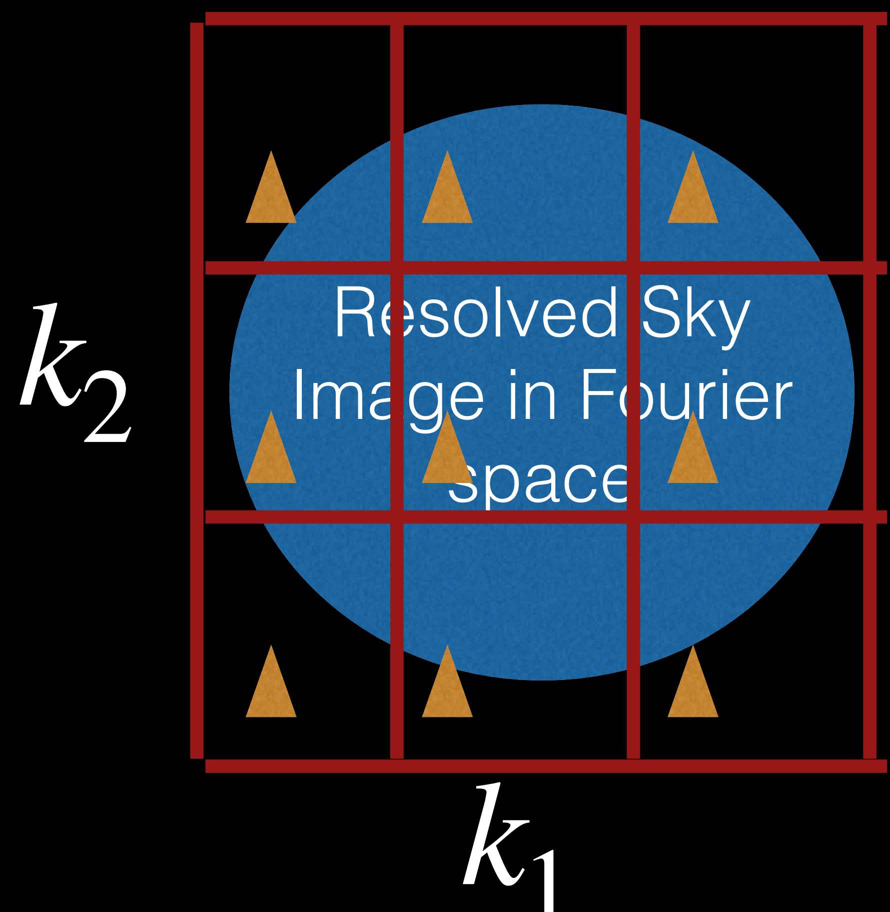
Image model in Fourier space - under-sampled images

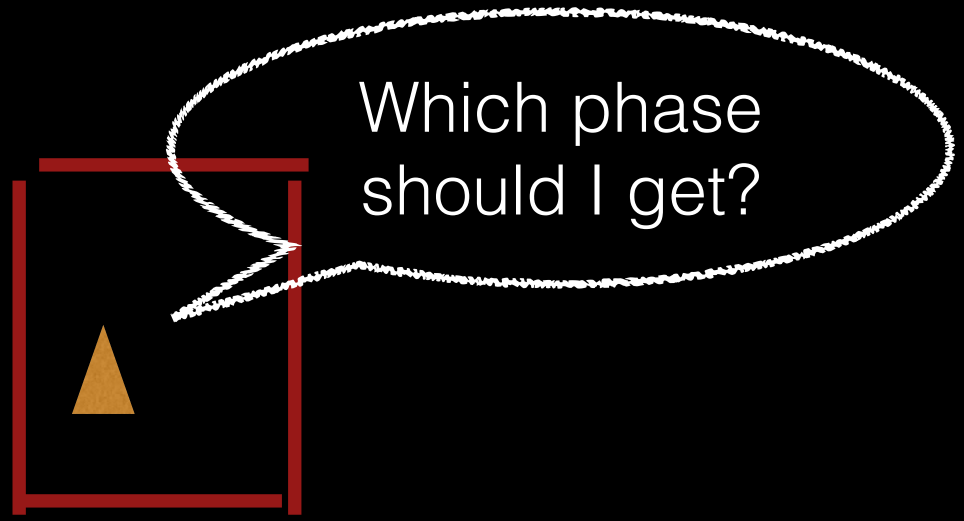


What breaks in the previous formalism?

- Shift and rotate operations not well defined
- Different triangles obtain different phases!

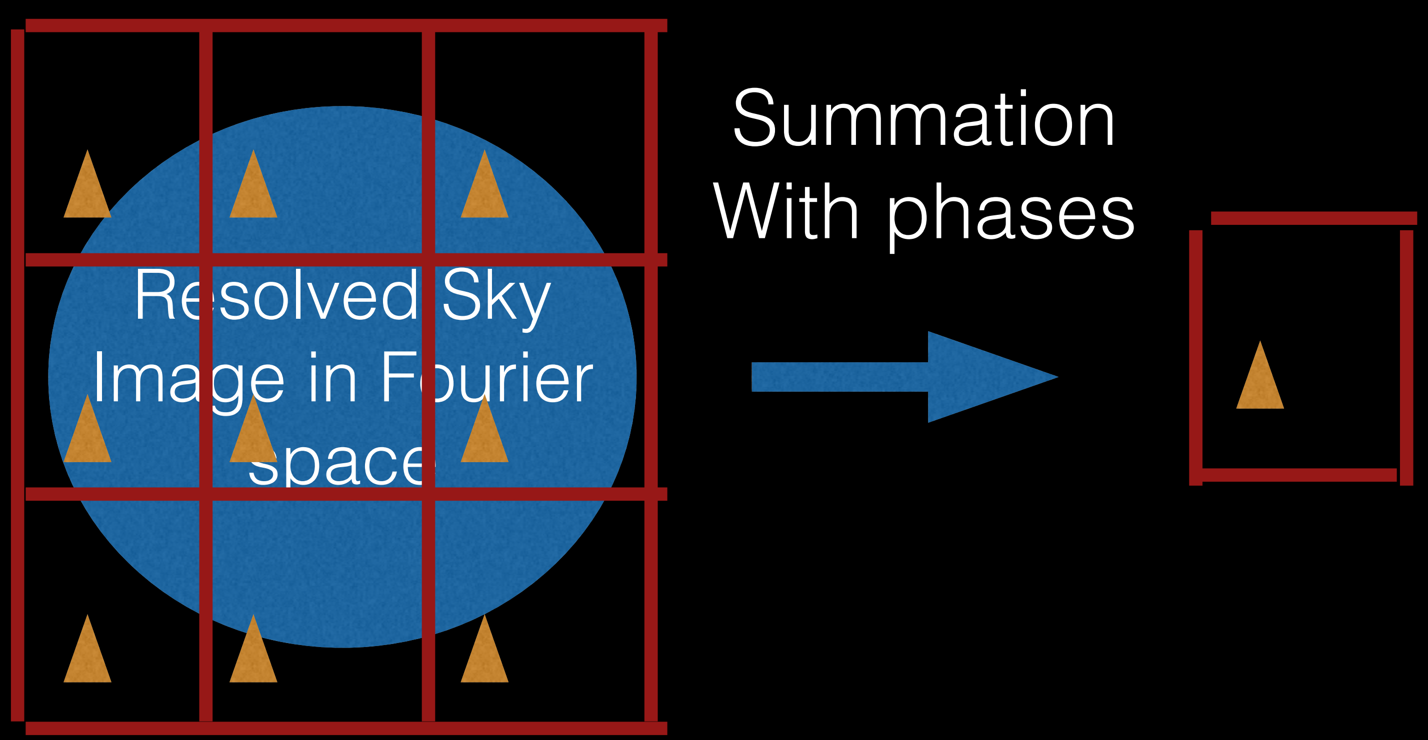
$$\hat{T}(k_1, k_2) \to \hat{T}(k_1, k_2)e^{i\Delta x \cdot k}$$





Tentative solution

- “Solve” many small systems of linear equations
- Use basis in which noise covariance is shift invariant (not response!)
- Output is an image with a “integer shift invariant”, and local noise correlation covariance with only the necessary dimensions.
- Allows performing any local measurement efficiently



Advancements in Image Subtraction

Subtraction - Notation

- Image model:

$$R = P_r \otimes T + \epsilon_r$$

$$N = P_n \otimes T + \epsilon_n$$

- R - Reference image
- N - New image
- Noise is white and Gaussian
- No assumption on T.

Problems with past image subtraction algorithms

- No argumentation or reasoning.
  - Reduced sensitivity
  - Unclear what further signal processing should be applied
- False positives.
  - Machine learning sifting of millions of candidates per day
  - Human scanning for final sifting stage.
    - No automatic followup + inevitable 1 hour latency.
- Numerically unstable.
- Slow (may be a serious constraint for large surveys).

Existing methods for image subtraction

- Phillips & Davis (95)

$$\widehat{D_{Phillips}} = \hat{N} - \frac{\hat{P}_n}{\hat{P}_r}\hat{R}$$

- Allard & Lupton (98)

$$D_{AL} = N - K \otimes R$$

- Bramich (2000)

- Gal-Yam et al. (06).

$$D_{GY} = P_r \otimes N - P_n \otimes R$$

Optimal transient detection

- Stating the hypotheses:

$$\mathcal{H}_0 : \; N = T \otimes P_n + \epsilon_n$$

$$\mathcal{H}_1 : \; N = (T + \delta(q)) \otimes P_n \epsilon_n$$

- Applying Neyman-Pearson:

$$S = \frac{\mathcal{P}(R, N|\mathcal{H}_1)}{\mathcal{P}(R, N|\mathcal{H}_0)} = \frac{\mathcal{P}(N|R, \mathcal{H}_1)}{\mathcal{P}(N|R, \mathcal{H})_0} \frac{\mathcal{P}(R|\mathcal{H}_1)}{\mathcal{P}(R|\mathcal{H}_0)} = \frac{\mathcal{P}(N|R, \mathcal{H}_1)}{\mathcal{P}(N|R, \mathcal{H})_0}$$

$$\hat{S} = \frac{|\hat{P}_r|^2 \overline{\hat{P}_n} \hat{N} - |\hat{P}_n|^2 \overline{\hat{P}_r} \hat{R}}{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}$$

Proper image subtraction

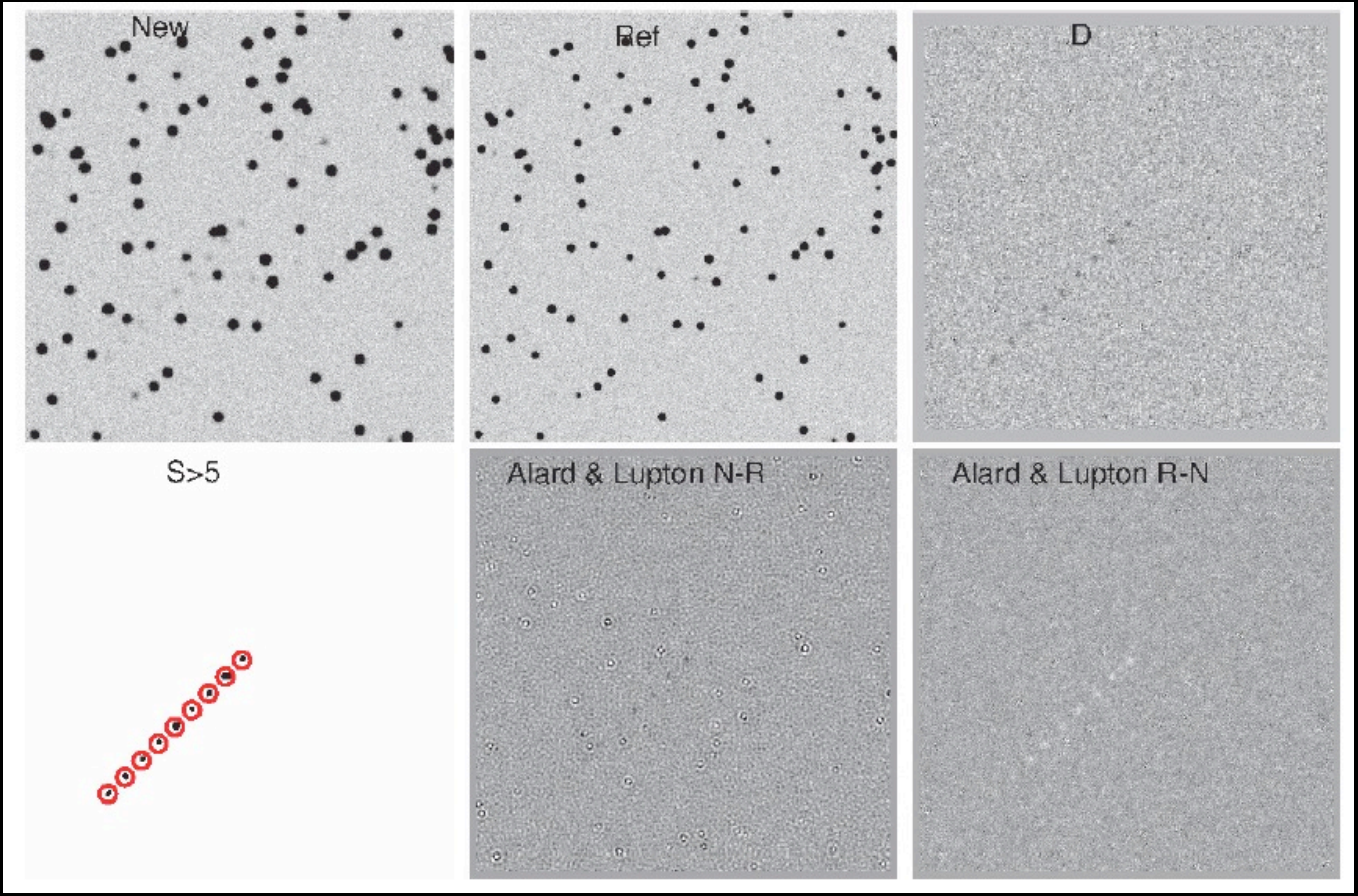
- What if we want to identify all types of transients?
  - Including defects and cosmic rays

$$\hat{D} = \frac{\hat{P}_r \hat{N} - \hat{P}_n \hat{R}}{\sqrt{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}} \qquad \hat{P_D} = \frac{\hat{P}_r \hat{P}_n}{\sqrt{\sigma_n^2 |\hat{P}_r|^2 + \sigma_r^2 |\hat{P}_n|^2}}$$

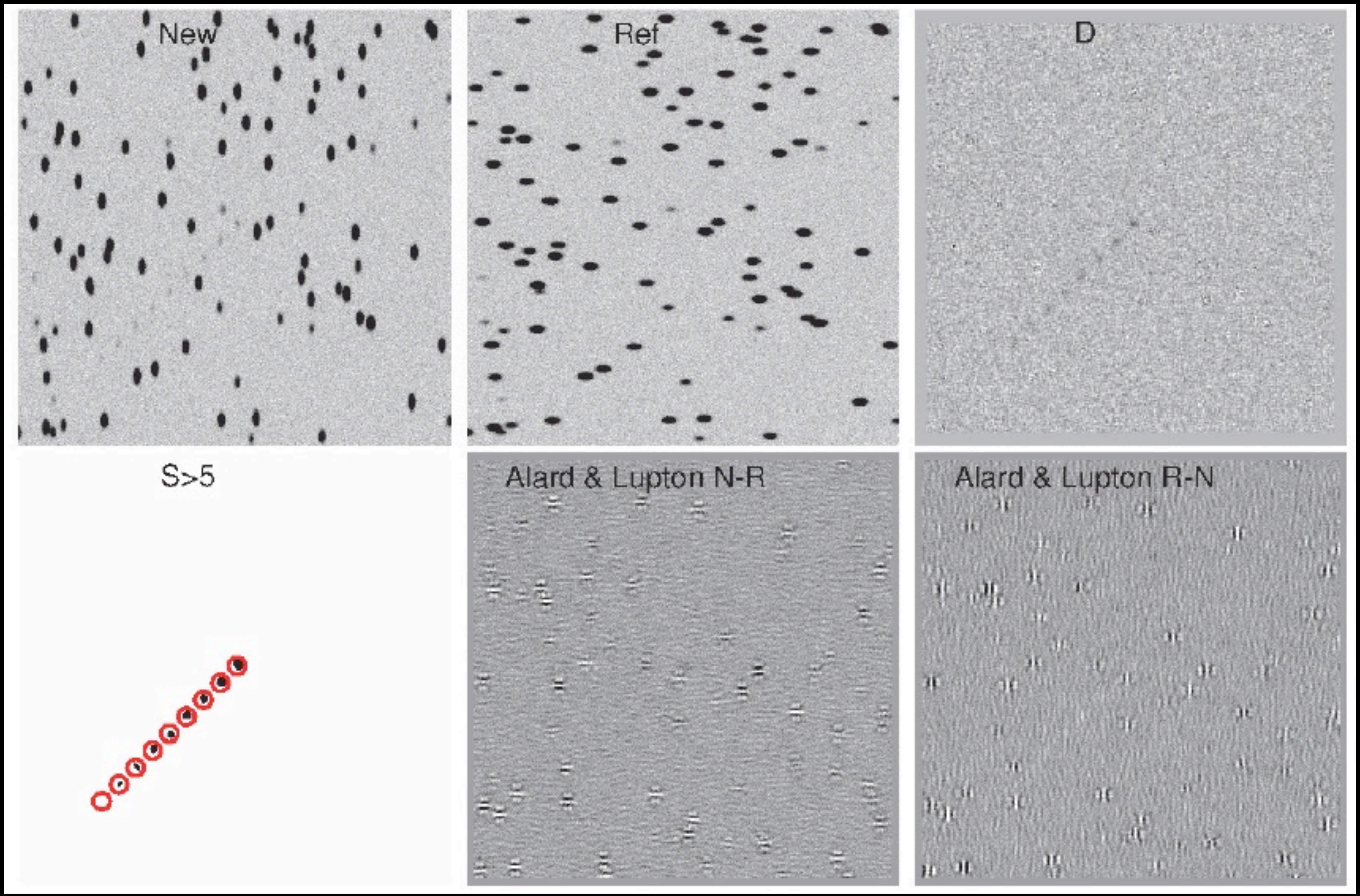
Properties of proper image subtraction

- Optimally sensitive (with a **rigorous mathematical proof**)
- Convolution kernels are **local (no problem with spatially varying PSFs).**
- **5%-50% more sensitive** than past methods**.**
- **Reliable significance and error bars** (this in itself increases sensitivity).
- Closed form (and symmetric to N,R interchange).
- Sufficient for testing/measuring **any difference** between the images.
- Fast and Numerically stable

Simulations - 1



Simulations - 2



Correcting for source noise and astrometric noise

- Can separate the transient detection score to the “New” part and “reference” part

$$S_n = N \otimes k_n \, , \; S_r = R \otimes k_r$$

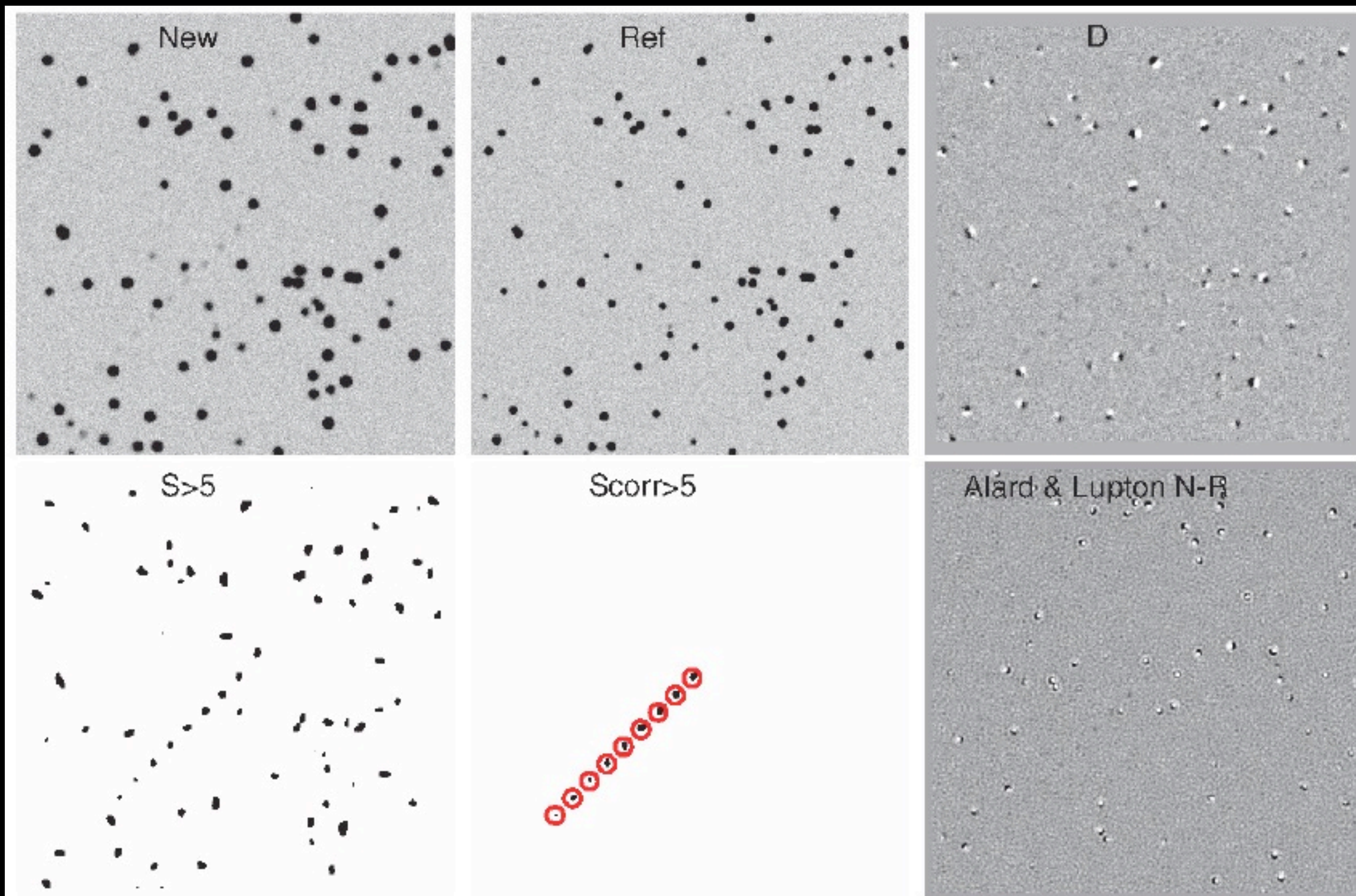
- Can bound the influence of source noise using the point-wise variance maps of N and R.

$$V_{S_n} = V(N) \otimes k_n^2 \, , \; V_{S_r} = V(R) \otimes k_r$$

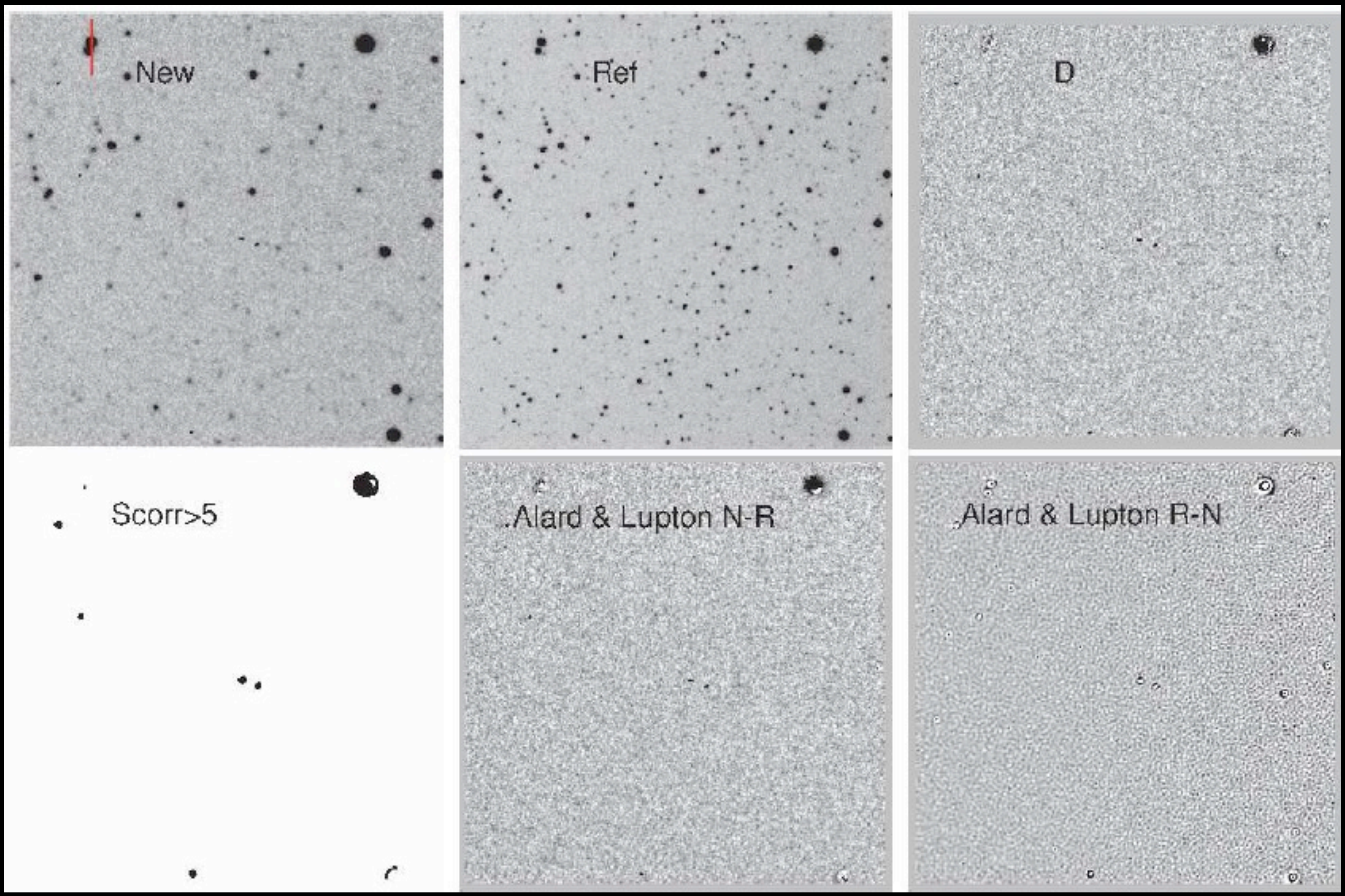
- Can bound the influence of astrometric noise using their pixel derivatives

$$V_a = \frac{\partial S_n}{\partial x} \sigma_{a_x}^2 + \frac{\partial S_n}{\partial y} \sigma_{a_y}^2$$

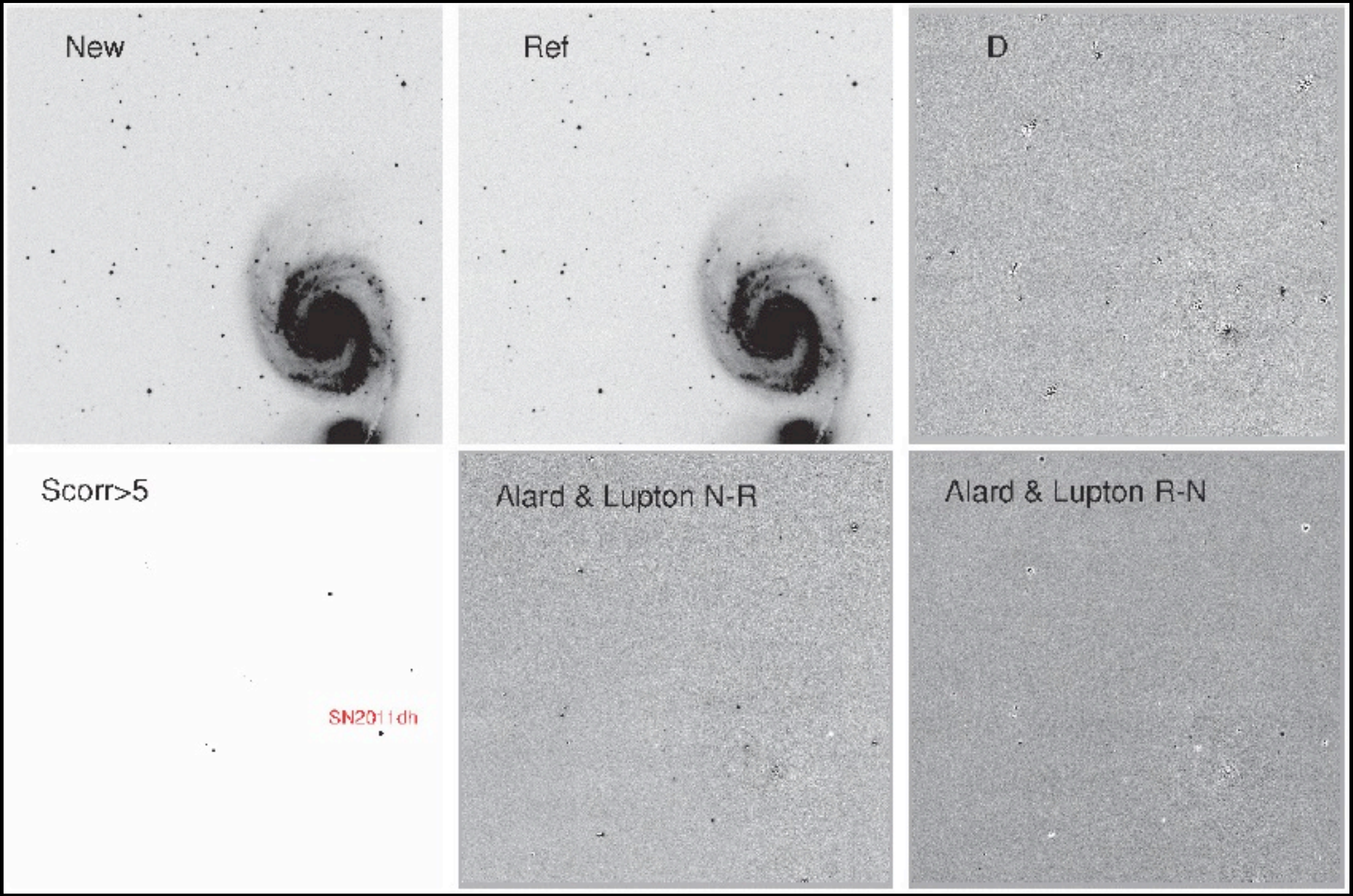
Correcting astrometric noise



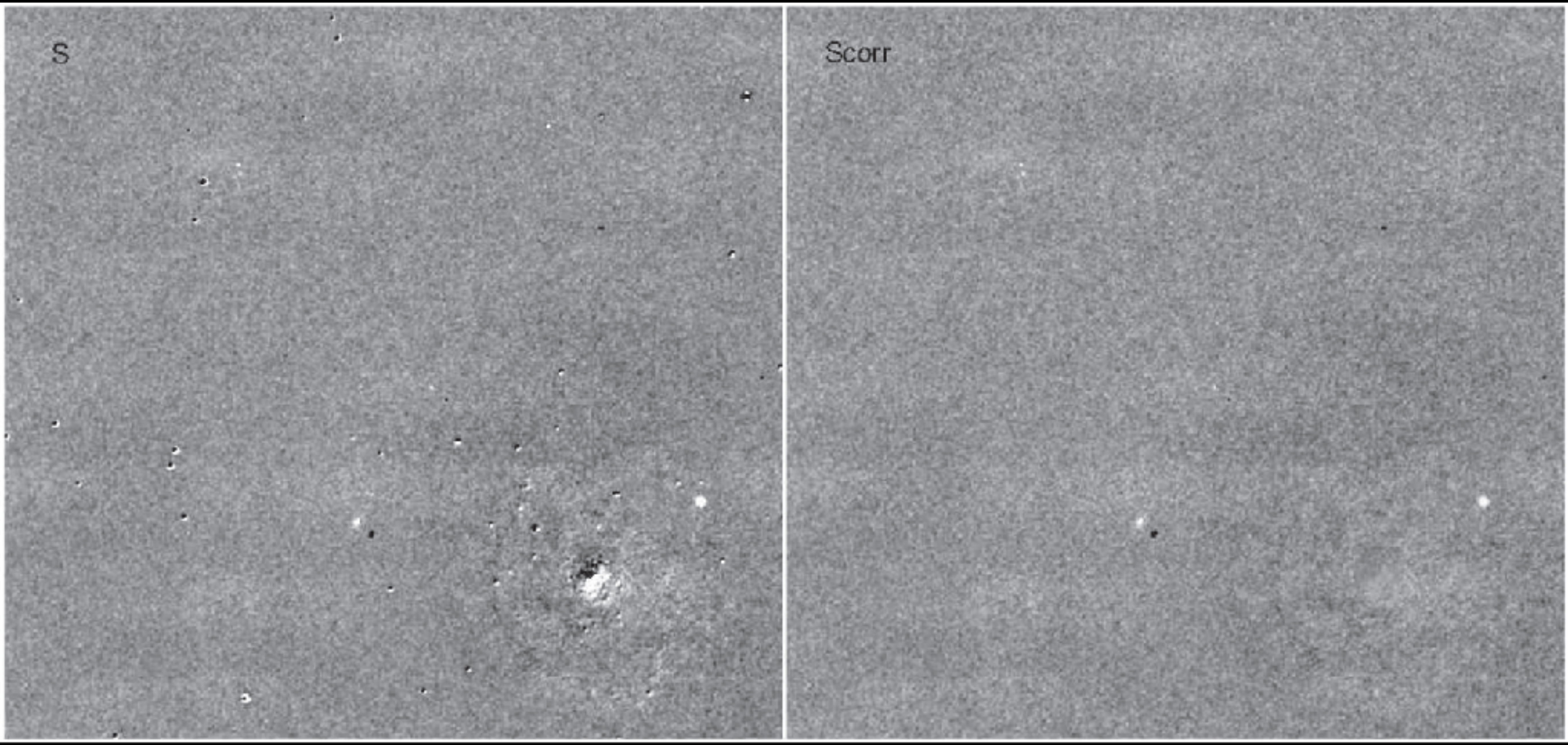
Real data - I



Real data - II

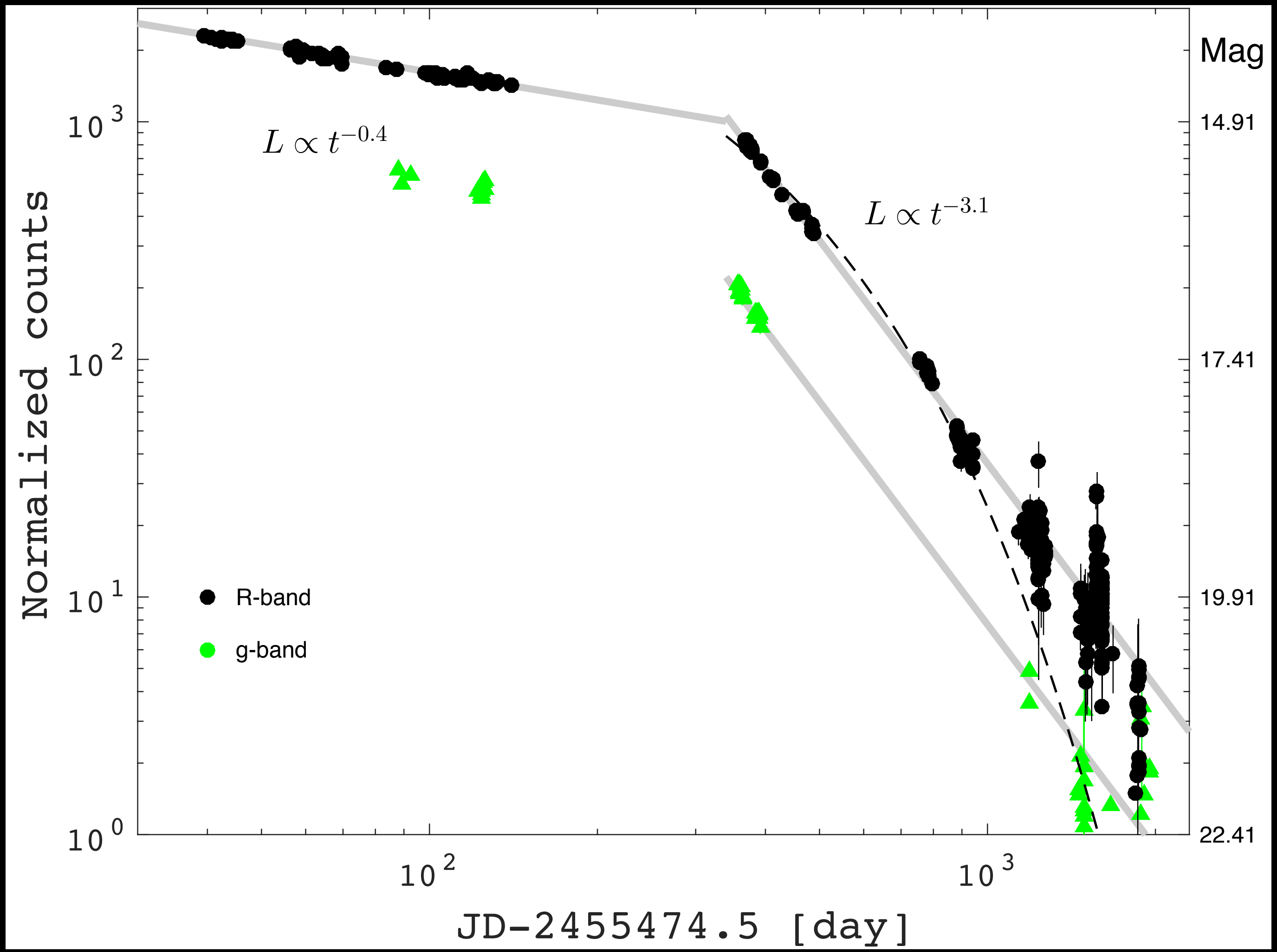


Real data - II continued

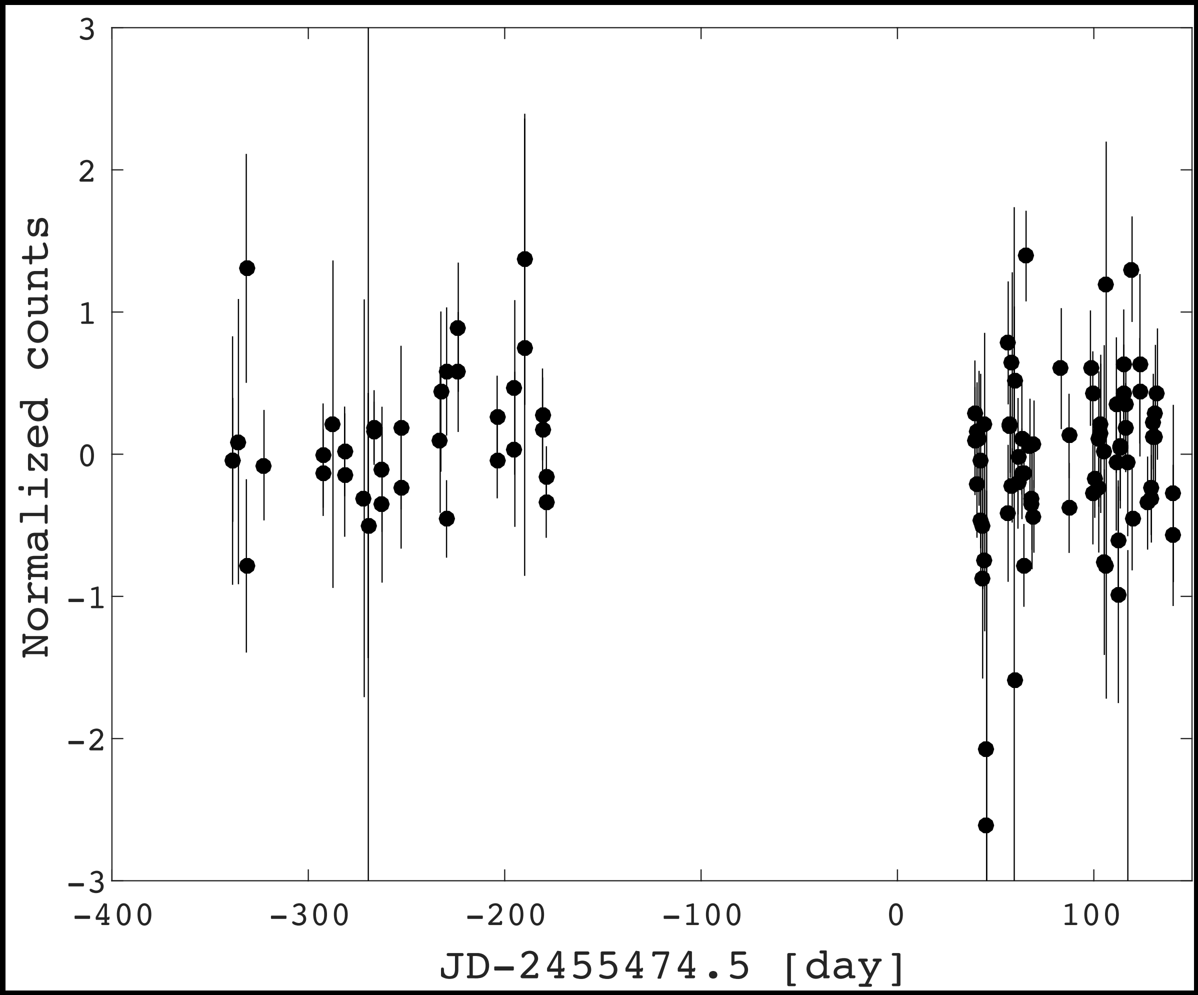


Real data - III

Subtracting a (very) bright galaxy



SN 2010jl light curve



Some random point

Current status

- The main subtraction algorithm in:
  - LSST, ZTF, BlackGEM
- At least 7 independent implementations
  - Non of them is "mine"
  - I can recommend https://github.com/pmvreeswijk/ZOGY
- Dramatically easier to detect TDE's
- ZTF now has **robotic, automated followup**

Future prospects:

Image subtraction with under-sampled images

- Given a set of **under sampled** reference images, and an under sampled new image ->
- Produce a sufficient statistic for any measurement on the image difference
  - Most importantly transient detection.
- Scientific target:
  - ULTRASAT
  - JWST (Let it launch!)

Questions?

- **Contact me** if you are a talented student/postdoc and you are:
  - Looking for a position (or to collaborate)
  - Interested in novel statistics and algorithms.
  - looking to make a dramatic impact in astrophysics.
- Come to me, and you will: **learn** a lot, **Invent and** have all **legitimacy** and **resources** required to **revolutionize** a field in astrophysics.
- There are special postdoc scholarships for Brazilian citizens in Weizmann!

I am hiring.